Large-Eddy Simulations of Baroclinic Instability and Turbulent Mixing

Eric D. Skyllingstad
College of Oceanic and Atmospheric Sciences, Oregon State University
104 COAS Admin. Bldg.
Corvallis, OR 97331
Phone: (541) 737-5697 Fax: (541) 737-2064 Email: skylling@coas.oregonstate.edu

Roger M. Samelson
College of Oceanic and Atmospheric Sciences, Oregon State University
104 COAS Admin. Bldg.
Corvallis, OR 97331
Phone: (541) 737-4752 Fax: (541) 737-2064 Email: rsamelson@coas.oregonstate.edu

Award Number: N00014-09-1-0268

LONG-TERM GOAL

The long-term goal of this project is to improve our ability to understand, model and predict lateral mixing and the associated submesoscale physical structure and processes in the upper and interior ocean.

OBJECTIVES

The main objective of this project is to examine the interaction between baroclinic, mesoscale eddies and turbulence using a large-eddy simulation (LES) model. Cases will focus on strong, baroclinic waves that form in the mixed layer along surface fronts with scales of a few km, and on mesoscale eddies that are imbedded within larger scale frontal regions. Our goal is to quantify, understand, and ultimately parameterize the physical processes that lead to lateral mixing. Simulations will help guide field experiments planned as part of the Lateral Mixing DRI, and provide a tool for understanding observations in the analysis phase of the project.

APPROACH

High-resolution simulations of baroclinic instability and the interaction of mesoscale flow with turbulent mixing are conducted and analyzed using a large-eddy simulation model. Our analysis centers on quantifying and understanding the mechanisms by which small-scale turbulent structure develops on the mesoscale field, the physical processes and balances that control lateral mixing of fluid properties across the unstable front, and the transition from strongly horizontal, geostrophic motion on the mesoscale to three-dimensional, quasi-isotropic, non-hydrostatic motion on turbulent scales.

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **30 SEP 2012**	2. REPORT TYPE	3. DATES COVERED **00-00-2012 to 00-00-2012**
4. TITLE AND SUBTITLE **Large-Eddy Simulations of Baroclinic Instability and Turbulent Mixing**		5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)		5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Oregon State University,College of Oceanic and Atmospheric Sciences,104 COAS Administration Building,Corvallis,OR,97331-5503**		8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSOR/MONITOR’S ACRONYM(S) 11. SPONSOR/MONITOR’S REPORT NUMBER(S)
12. DISTRIBUTION/AVAILABILITY STATEMENT **Approved for public release; distribution unlimited**		
13. SUPPLEMENTARY NOTES		
14. ABSTRACT		
15. SUBJECT TERMS		

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**5**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

WORK COMPLETED

Research during the fouth year of this project has focused on completing a publication focused on simulations of baroclinic instability and the generation of turbulence. Further research on the transfer of energy between baroclinic scales and turbulence scales was also conducted and is reported here.

RESULTS

A key question in our analysis of small-scale baroclinic instability concerns the fate of kinetic energy generated at baroclinic scales, which can be removed from the system by either a gradual cascade of energy through progressively smaller scale, quasi-two-dimensional eddies or through direct production along frontal boundaries of three dimensional turbulence that decays via sub-grid-scale viscous dissipation. The structure of kinetic energy spectra as a function of horizontal wave number magnitude $k = \sqrt{k_x^2 + k_y^2}$ for the current simulation (from a two-dimensional spectral decomposition, averaged over wavenumber vectors of approximately equal magnitude, and then divided by k to obtain the spectral density) also suggests that energy at large scales is not removed by a continuous inertial cascade process: the spectral slope is about -3, more indicative of an energy-conserving enstrophy-cascade than an energy-cascade regime, and there is enhanced energy and a nearly flat spectrum at the largest wavenumbers, between scales of 6 and 60 m (Fig. 1). A rough estimate of the dissipation spectra may be obtained by multiplying the kinetic energy spectra (Fig. 1) by k^2, suggesting a modest peak at larger wavenumbers $k > 0.1$ m^{-1}; the corresponding variance-preserving form, obtained by multiplying by a third factor of k, would indicate that the dominant contribution to the integrated dissipation is from wavenumbers $k > 0.1$ m^{-1}, or at scales smaller than 60 m but still well-resolved by the 3 m grid spacing.

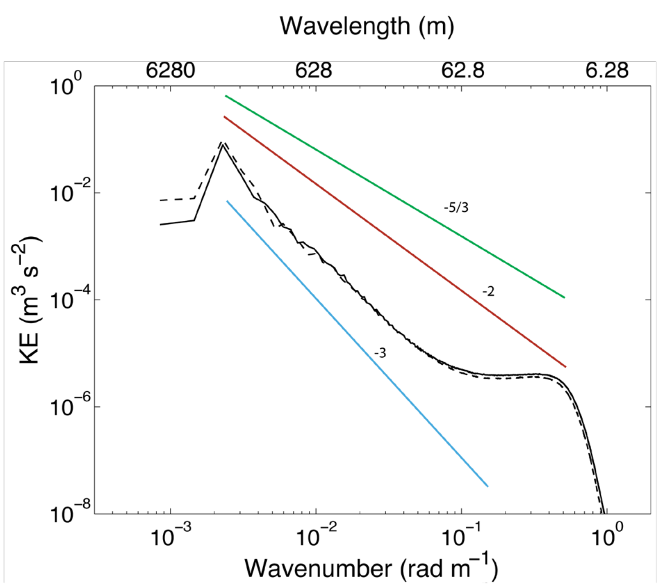

Figure 1. Two-dimensional KE spectra calculated as described in the text. The solid line is from hour 84 and dashed line is from hour 96. Colored lines denote various spectral slopes as labeled.

A more complete description of the spectral scale interactions can be obtained through a spectral decomposition of the kinetic energy balance (Frisch, 1995; Capet et al., 2008; Molemaker et al. 2010),

$$\hat{u}'^{*}_{i} \cdot \frac{\partial \hat{u}'_{i}}{\partial t} = \hat{u}'^{*}_{i} \cdot \widehat{u_j \frac{\partial u_i}{\partial x_j}} + \hat{u}'^{*}_{i} \cdot \widehat{\nabla p} - \hat{u}'^{*}_{3} \hat{\delta}_{i3} g \frac{\hat{\rho}'}{\rho_o} - \hat{u}'^{*}_{i} \cdot \frac{\partial \widehat{\left(\overline{u''_i u''_j}\right)}}{\partial x_i} \tag{1}$$

where $\hat{u}\left(x, k_y, z\right)$ denotes a 2-dimensional spectrum in the horizontal direction, asterisk denotes complex conjugate, and terms are defined in order as storage term, energy flux divergence, pressure term, buoyancy term and dissipation term. For this analysis, the perturbation velocity and the terms that enter in (1) from the right-hand side of the momentum balance were computed and integrated vertically over the entire model domain, and separately averaged temporally over a 5 minute time period, with the product then providing snapshots of the spectral energy balance at two different model times, hours 84 and 96, representing peak wave growth and reduced growth with established ring or eddy circulation features, respectively (Fig. 2). The storage term in (1) was calculated by computing the difference in spectral energy over the 5-minute averaging period and used to estimate the budget residual.

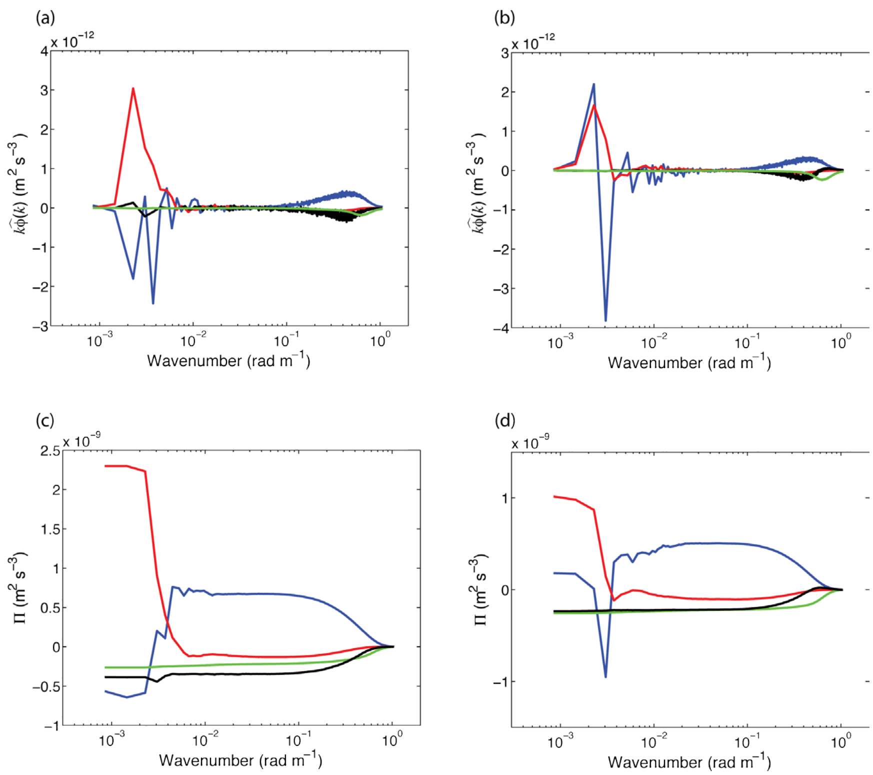

Figure 2. Domain averaged spectral components of the (a,b) kinetic energy budget and (c,d) spectral energy flux for hours (a,c) 84 and (b,d) 96. The energy flux divergence term is blue, buoyancy term red, dissipation term green, and residual is black. Spectral components in (a,b) are multiplied by wavenumber k to preserve variance.

The integrated spectral fluxes,

$$\Pi(k) = \int_{k}^{k_{\max}} \phi(k)\,dk\,, \tag{2}$$

where $\phi(k)$ are the individual terms in (1), provide a measure of the direction of energy transfer by each term in the budget. For example, positive values for the integrated energy flux divergence term indicate a forward transport for larger wavenumbers, with energy removed by dissipation and numerical effects at small scales (Fig. 2). The relatively constant values for the dissipation and residual for $k < 0.1$ m^{-1} indicate that energy loss from these terms occurs only at the small scales ($k >$ 0.1 m^{-1}).

Large-scale baroclinic waves ($k < 0.01$ m^{-1}) continue to gain kinetic energy from buoyancy production generated through the slumping of the mixed layer front through hour 96 (Fig. 2a,b). At hour 84, large-scale eddy energy is lost through the energy flux divergence term, even though the overall kinetic energy is still increasing. As the eddies form closed circulations at hour 96, the energy flux divergence term acts to redistribute kinetic energy upscale as shown by the negative values for $k \sim 0.003$ m^{-1} and positive values for $k \sim 0.002$ m^{-1}. Baroclinic modes clearly dominate the energy balance at large scales while turbulence formation and dissipation at small scales close the budget. There is almost no indication of significant energy production or dissipation at wavenumbers between 10^{-2} m^{-1} and 10^{-1} m^{-1}. (Fig. 2a,b). This result could be misinterpreted as an indication that an inertial subrange exists between the baroclinic modes and turbulence at the dissipation scales. However, the physical structure of the flow, with energetic turbulence appearing only at intensified fronts, and the reduced strength of eddies at intermediate scales indicated by the steep spectral slope (Figure 1) suggest that turbulence is gaining significant energy primarily through non-local spectral transfer rather than through a continuous forward cascade.

The spectral energy budget (8) at turbulence scales (Fig. 2a,b, $k > 0.1$ m^{-1}) shows a peak in energy flux divergence associated with small-scale instabilities in the frontal region. At these scales, perturbation kinetic energy is lost via negative buoyancy production when vertical mixing destroys stratification associated with the sloping fronts and the mixed layer base. Dissipation is a maximum near the grid scale at about $k = 0.8$ m^{-1}. However, because the spectral decomposition (1) of the energy budget is not fully compatible with the numerical discretization and because the model grid spacing is relatively coarse for simulating turbulent eddies, there is also a fractional residual that is relatively large at these small scales, where the budget terms are all small. The residual in the total kinetic energy budget is negligible. The flow-dependent turbulence closure in the LES model allows relatively large velocity fluctuations at small scales, with a steep roll-off of the kinetic energy spectrum just above the grid scale (Fig. 1). This evidently exaggerates the incompatibility between the spectral decomposition (1) and the discretization, leading to a larger residual near the grid scale than may be seen in hydrostatic circulation models that spread viscous dissipation over a wider range of larger scales, such as the simulations described by Capet et al. (2008). Note in this context that a related incompatibility is recognized by Capet et al. (2008), who argue that their near-zero model shear-production term can be separated ex post facto, by an alternative numerical discretization, into a positive production term and a balancing negative term arising from the model discretization that is presumed to represent a physical dissipation. In any case, our main conclusion is that the transfer of energy from the larger, baroclinic modes to small-scale turbulence is a more direct process that is less dependent on a cascade of energy than would be the case for a classical shear or buoyancy driven turbulent boundary layer.

REFERENCES

Capet, X., J. C. McWilliams, M. J. Molemaker, and A. F. Shchepetkin, 2008: Mesoscale to submesoscale transition in the California Current System. Part III: Energy Balance and Flux. *J. Phys. Oceanogr*., **38**, 2256-2269.

Frisch, U., 1995. *Turbulence*. Cambridge University Press, New York, 296 pp.

Molemaker, M. J., J. C. McWilliams, and X. Capet, 2010: Balanced and unbalanced routes to dissipation in an equilibrated Eady flow. *J. Fluid Mech*., **654**, 35–63.

PUBLICATIONS

Skyllingstad, E. D. and R. M. Samelson, 2011. Baroclinic Frontal Instabilities and Turbulent Mixing in the Surface Boundary Layer. Part I: Unforced, J. Phys. Oceanogr., 1701-1716.